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NORMAL MOMENTUM TRANSFER STUDY BY A DYNAMIC TECHNIQUE

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A novel dynamic technique for normal momentum accommodation study in a "gas–solid body" system is developed. The method can also be used for pressure measurements and adsorption studies. The experimental setup is described. The data obtained are interpreted with allowance for geometric parameters. The normal momentum accommodation coefficients are estimated on the basis of measurement results in systems "hydrogen (nitrogen) gas mixture–single-crystal silicon".

Key words: *rarefied gas, gas–surface interaction, normal momentum, accommodation coefficient.*

Introduction. For a long time, dynamic pressure gauges [1, 2], in particular, those based on the principle of mechanical oscillations of an elastic plate in a gas, have not been widely used in experiments, which was caused by the complexity of fabricating sensors possessing stable elastic properties and providing necessary resolution of the measured parameters. Achievements in the field of the MEMS technology favored a certain progress in this direction.

Characteristics of dynamic pressure gauges allow their application in various fields of science and engineering. For example, by measuring the frequency shift of sensor oscillations, one can measure gas adsorption on the sensor surface. Another important application of the dynamic technique is the investigation of momentum-transfer efficiency in the course of interaction of the gas and solid surface, which is generally characterized by accommodation coefficients [3]. The values of accommodation coefficients, in particular, the normal momentum accommodation coefficient (NMAC), are mainly obtained by the molecular beam technique [4, 5]. Implementation of this method, however, faces significant difficulties including the problem of interpreting the results and using them to describe the processes of gas–surface interaction under weakly nonequilibrium conditions.

The proposed method for NMAC measurement has significant advantages over other methods; one advantage is the simplicity of its implementation. As for interpretation of the data obtained, it is also rather simple if some assumptions are used in a particular experimental situation.

The present paper offers a brief description of the dynamic technique and its implementation to investigating the transfer of normal momentum of rarefied gas molecules upon interaction with the solid body surface. Geometric factors that can affect interpretation of the data obtained are analyzed. For a particular case, the normal momentum accommodation coefficients are estimated on the basis of measurement data.

Experimental Technique. The essence of the technique proposed is the measurement of a force acting on a plate moving in a gas. The plate motion can be ensured by free oscillations.

In the present work, we measured the damping coefficient of free linear oscillations of an elastic plate in the free-molecular regime with varied gas pressure.

The measurement path of the experimental setup is schematically shown in Fig. 1. The sensor is fabricated from a plate of microcontoured single-crystal silicon by the MEMS technology. This allowed an almost perfect attachment of the elastic part of the plate 5×5 mm and obtaining rather stable damping coefficients of plate oscillations $\beta = (0.020 \pm 0.002)$ sec^{-1} under conditions of ultrahigh vacuum (of the order of 10^{-7} Pa). The thicker part of the plate, which does not directly participate in the momentum-transfer process, was fixed by assembling screws used in ultrahigh vacuum engineering. The elastic plate together with the fixed electrode form an electric

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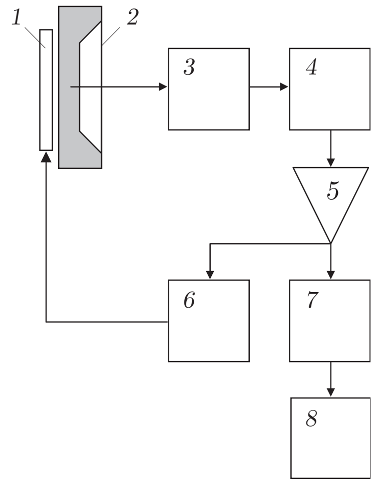

Fig. 1. Measurement path of the experimental setup: 1) fixed electrode; 2) microcontoured silicon plate; 3) high-frequency LC generator; 4) frequency detector; 5) selective amplifier; 6) oscillation-generation system; 7) analog-to-digital converter; 8) computer.

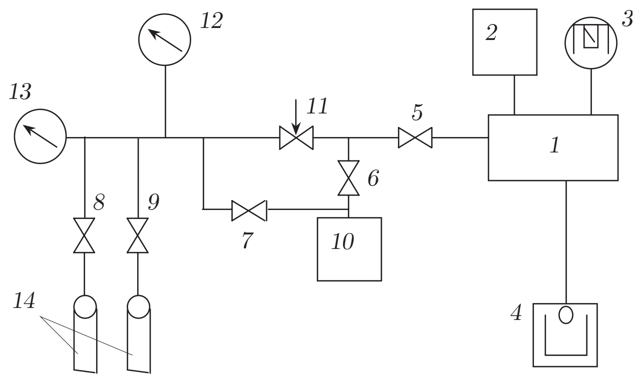

Fig. 2. Vacuum chamber and gas-inlet system: 1) vacuum chamber; 2) monopolar mass spectrometer; 3) vacuum pressure gauge; 4) sputter-ion pump; 5–9) vacuum valves; 10) backing pump system; 11) gas-inlet valve; 12) optical membrane gauge; 13) thermocouple vacuum gauge; 14) gas reservoirs.

capacitor of about 15 pF, which is connected to the oscillating LC generator with a resonance frequency of about 10 MHz. The initial oscillations of the elastic plate are excited by the electric field between the sensor and fixed electrode by means of a specially designed system of self-induced oscillations. The excitation system serves as a chain of feedback and a source of external energy. The natural oscillation frequency of the elastic plate is about 1250 Hz.

The microcontoured silicon plate and the fixed electrode are mounted vertically in the vacuum chamber, which is shown in Fig. 2 together with the gas-inlet system. The gas is evacuated from the vacuum chamber by a sputter-ion pump with a pumping velocity of 100 liters/sec. The residual pressure in the vacuum chamber is measured by a Penning vacuum gauge and reaches less than 10^{-7} Pa. The gas composition is controlled by a monopolar mass spectrometer for pressures lower than 10^{-3} Pa. The vacuum chamber is connected to the gas-inlet system through a valve. Preliminary evacuation is performed by the backing pump system consisting of two adsorption pumps and a mechanical vacuum pump. The gas pressure in the inlet system is measured by an optical membrane gauge and a thermocouple vacuum gauge.

Measurement of the damping coefficient of free linear oscillations of the elastic plate with varied gas pressure reduces to registration of the value of a capacitor formed by the elastic plate and the fixed electrode and included into the oscillating circuit of the LC generator.

For NMAC determination, the measurements are performed with fixed values of gas pressure. Except for the standard procedure of cleansing after the final stage of fabrication, the sensor surface received no other treatment.

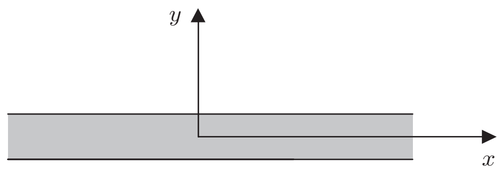

Fig. 3

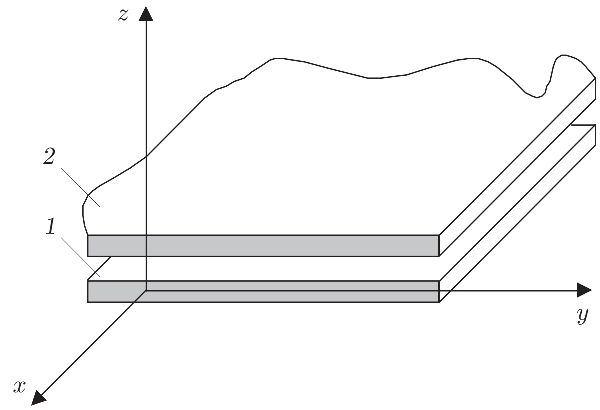

Fig. 4

Fig. 3. Geometry of the problem of an isolated plate.

Fig. 4. Geometry of the problem for two infinite parallel plates.

Normal Momentum Transfer to an Isolated Infinite Surface. We consider the problem of oscillations of an isolated plate (sensor) in a rarefied gas without taking into account the influence of the fixed electrode located near the plate [6, 7].

A thin infinite plate is immersed into an isothermal rarefied gas. The "gas–solid body" system is in a quasi-equilibrium state. The plate velocity u is normal to the surface (in the y direction) and significantly lower than the mean heat velocity of gas molecules v (Fig. 3).

The normal momentum flux transferred to the surface is

$$|P_n| = |P_{ni}| + |P_{nr}|,$$

where P_{ni} and P_{nr} are the normal momentum fluxes of incident and reflected molecules, respectively.

The incident gas molecules are assumed to have the Maxwellian velocity distribution. The normal momentum flux for reflected molecules is

$$|P_{nr}| = (1 - \alpha_n)|P_{ni}| + \alpha_n|P_{nw}|,$$

where P_{nw} is the normal momentum flux of reflected molecules in thermal equilibrium with the surface and α_n is the NMAC in Knudsen's definition:

$$\alpha_n = \frac{|P_{ni}| - |P_{nr}|}{|P_{ni}| - |P_{nw}|}.$$

The number densities n of incident and reflected molecules are equal and independent of coordinates.

The normal momentum flux transferred to the surface is determined as

$$P_n = (2 - \alpha_n)nmuv$$

(m is the molecular weight). With allowance for the mean velocity of molecules, the resultant force can be written as

$$F = 2(2 - \alpha_n)p(2m/(nkT))^{1/2}u,$$

where p and T are the gas pressure and temperature, respectively.

The result obtained forms the basis for NMAC calculation from experimental data.

If the plate performs free normal linear oscillations and the assumptions of the elasticity theory are satisfied, the corresponding equation takes the following form [8]:

$$\frac{\partial^2 U}{\partial t^2} + 2\beta\,\frac{\partial U}{\partial t} + \frac{D}{\rho h}\,\Delta^2 U = 0. \tag{1}$$

Here U is the displacement coordinate (hence, the velocity is $u = \partial U/\partial t$), ρ is the plate-material density, h and D are the plate thickness and hardness, β is the damping coefficient of free oscillations, and Δ is the Laplacian.

The second term in Eq. (1) is the force related to dissipative processes. In our case, the damping coefficient is represented as the sum of two coefficients: β_s and β_i. The coefficient β_s characterizes the gas–surface interaction, and the coefficient β_i refers to dissipative processes in the plate itself. In this case, the dependence of the damping coefficient of elastic oscillations of the plate in the gas on the NMAC acquires the form

$$\beta_s = \frac{2-\alpha_n}{\rho h} p \sqrt{\frac{2m}{\pi k T}}. \tag{2}$$

Expression (2) can be used for experimental data processing.

Normal Momentum Transfer Between Two Infinite Plates. Under real conditions, the plate sensor and the electrode form a kind of a channel, the rigorous solution requires consideration of a system of two plates (Fig. 4). The lower plate 1 performs normal oscillations. The velocity of plate motion is significantly lower than the mean heat velocity of gas molecules. The distance between the plates is significantly smaller than the mean free path of gas molecules. The gas state above plate 2 and below plate 1 is described by the Maxwellian velocity-distribution function.

The boundary conditions for this problem follow from Knudsens' definition of the NMAC. The origin of the Cartesian coordinate system is chosen on the lower oscillating plate 1. The upper plate is motionless and corresponds to the second electrode of the capacitor included into the oscillating circuit of the LC generator.

The following assumptions are made. The reflected molecules have the Maxwellian velocity-distribution function. The number of intermolecular collisions is negligibly small as compared to the number of collisions of gas molecules with the surface.

The normal momentum fluxes for molecules reflected from inner surfaces of the plates can be written as

$$|P^1_{nr}| = (1-\alpha^1_n)|P^1_{ni}| + \alpha^1_n |P^1_{nw}|, \qquad |P^2_{nr}| = (1-\alpha^2_n)|P^2_{ni}| + \alpha^2_n |P^2_{nw}|.$$

According to the problem geometry, the relations between the main fluxes are

$$|P^2_{nr}| = |P^1_{ni}|, \qquad |P^1_{nr}| = |P^2_{ni}|.$$

After appropriate substitutions, we obtain the system of equations

$$|P^1_{nr}| = (1-\alpha^1_n)(1-\alpha^2_n)|P^1_{nr}| + (1-\alpha^1_n)\alpha^2_n |P^2_{nw}| + \alpha^1_n |P^1_{nw}|,$$

$$|P^2_{nr}| = (1-\alpha^2_n)(1-\alpha^1_n)|P^2_{nr}| + (1-\alpha^2_n)\alpha^1_n |P^1_{nw}| + \alpha^2_n |P^2_{nw}|,$$

from which we find the total normal component of momentum imparted to the plate:

$$P^1_{ni} + P^1_{nr} = \frac{2\alpha^1_n \alpha^2_n}{\alpha^1_n + \alpha^2_n + \alpha^1_n \alpha^2_n}(P^1_{nw} + P^2_{nw}).$$

Further consideration of the problem requires a number of substantial simplifications, which inevitably lead either to an inadequate description of the real picture or to results similar to those obtained above for the isolated plate geometry. Otherwise, the analytical expressions become so cumbersome that experimental data processing becomes almost impossible.

Measurement Results. It is well known that an important process under conditions of ultrahigh vacuum is outgassing, which may significantly affect the composition of the medium under study. Since the outgassing velocity depends on time, one can measure the damping coefficient of plate oscillations with different total pressures and gas-phase compositions and then determine the NMAC.

The measurements were performed under conditions of escaping of residual gases in the vacuum chamber for different positions of valve 5 and nonoperating sputter-ion pump (see Fig. 2), which ensured formation of different gas mixtures. The mixtures examined contained 96% of nitrogen in one case and 62% of hydrogen in the second case. The additives were argon (4%) in the first case and helium (13%), water (11.6%), nitrogen (12.2%), and argon (1.2%) in the second case. The damping coefficient as a function of pressure for two different mixtures is plotted in Fig. 5 (P_0 is the pressure at which the measurements are started and P is the current gas pressure).

To determine the NMAC from Eq. (2), one has to know the absolute pressure of the gas. Since it is rather difficult to measure the absolute pressure under high-vacuum conditions, we used a mass spectrometer to measure the relative changes in partial pressures of the components.

The free-molecular regime was ensured by gas-pressure variation within 10^{-3}–10^{-4} Pa with rather low oscillation amplitudes of the elastic plate (less than 10^{-3} mm).

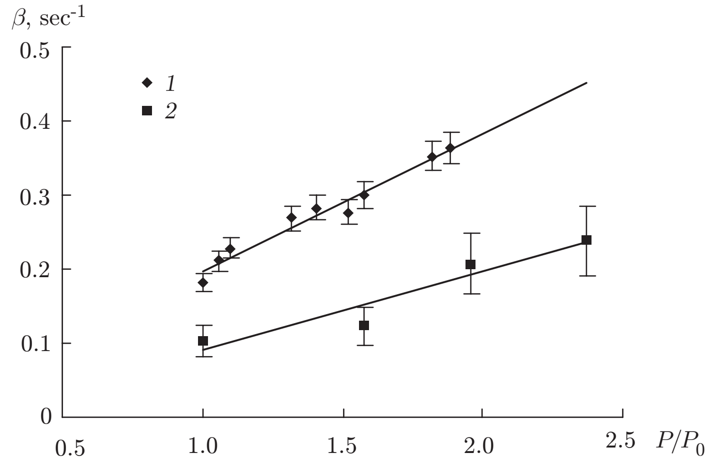

Fig. 5. Damping coefficient β as a function of gas pressure for the nitrogen mixture (points 1) and hydrogen mixture (points 2).

An analysis of experimental results (Fig. 5) allows us to conclude that the relative measurement error of the damping coefficient is approximately 10%, and the coefficient itself depends linearly on pressure. The slopes of the straight lines for two gas mixtures in Fig. 5 are significantly different, which is caused by the difference not only in the effective molecular weight of gas mixtures but also in NMAC values.

The measurement results are satisfactorily described by Eq. (2) derived for the isolated-plate geometry (linear dependence of β on p).

For a mixture of gases, Eq. (2) can be written as

$$\beta_s = \frac{2-\alpha_n}{\rho h}\, p\sqrt{\frac{2m_{\text{eff}}}{\pi kT}},$$

where m_{eff} is a certain effective molecular weight of the gas mixture.

According to additivity of the force entering into the second term of Eq. (1), the effective molecular weight of the gas mixture can be determined as

$$m_{\text{eff}} = \sum_i (x_i\sqrt{m_i})^2,$$

where x_i is the concentration of the corresponding component of the gas mixture.

A regression analysis of the damping curves (Fig. 5) yields

$$(2-\alpha_n^1)/(2-\alpha_n^2) = 0.78 \tag{3}$$

(α_n^1 and α_n^2 are some effective NMACs for the nitrogen and hydrogen mixtures, respectively).

Taking into account that the NMAC varies within 0 to 1, we can evaluate the ranges of variation of Eq. (3):

$$1/2 < (2-\alpha_n^1)/(2-\alpha_n^2) < 2. \tag{4}$$

The experimentally obtained ratio of NMAC values entering into Eq. (3) is within the range of admissible values of relation (4). In addition, in experiments whose results are described in [4, 5], the NMAC values for nitrogen and argon are within 0.7 to 1.0. Then, the NMAC value for a gas mixture with a high content of hydrogen will be expected to be from 0.33 to 0.71. This result does not contradict the known data [4].

It is difficult to estimate the NMAC more exactly because of the above-mentioned reasons related to absolute gas pressure measurement under conditions of high vacuum and to allowance for actual geometry of the experimental setup. The problem in comparing NMAC values is also related to the fact that NMACs in most cases were measured under significantly nonequilibrium conditions, which are observed in experiments with molecular beams. In addition, the physicochemical conditions on the "gas–solid body" interface can differ substantially from one experiment to another.

Conclusions. Thus, the dynamic technique is applicable to studying the transfer of the normal momentum component in a "gas–solid body" system under conditions of high vacuum. The NMAC estimates show that the results obtained are in good agreement with available experimental data.

The geometric parameters of the problem are theoretically analyzed to obtain analytical equations for interpreting experimental data.

Consideration of the problem of isolated plate oscillations in a rarefied gas with neglected influence of the fixed electrode yields satisfactory results. We believe that the method considered will find application in solving other physical problems.

Nevertheless, there are some problems associated with obtaining absolute NMAC values. These problems can partly be solved by changing the structure of the experimental setup and using numerical simulation results.

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